

Numerical investigation & optimisation of a small-scale travelling-wave thermoacoustic Stirling cryocooler

H Butson¹, M Gschwendtner¹, A Caughley², R Badcock³, G Lumsden³

¹ Auckland University of Technology, 55 Wellesley St E, Auckland 1010, New Zealand

² Callaghan Innovation, 5 Sheffield Crescent, Christchurch 8053, New Zealand

³ Paihau-Robinson Research Institute, PO Box 33436, Lower Hutt 5046, New Zealand

Email: holly.butson@aut.ac.nz

Abstract. Thermoacoustically driven cryocoolers have been an area of significant interest due to the driving mechanism being heat, which hence eliminates the need for moving mechanical parts and thus increases simplicity and reliability. A small-scale travelling-wave thermoacoustic Stirling cryocooler of this nature was designed and optimised in the thermoacoustic modelling software 'DeltaEC.' The cryocooler consisted of a resonator connected to a looped tube containing two thermoacoustic cores: one which functioned as an engine to produce acoustic work (TASE), and one as the cooler that converted this acoustic work to thermal energy to produce cooling (TASC). The purpose of the project was to investigate whether a small-scale version could be feasible and how the geometry could be optimised to improve its performance. Parameter studies focusing on the mesh number and wire diameter for the regenerators as well as the plate spacing and length for the heat exchangers were conducted to explore how they affect the cooling capacity and coefficient of performance (COP) of the machine. The results showed that for the TASC increasing the wire diameter decreases the cooling capacity for all mesh numbers but increases the COP for mesh numbers below 200, whilst for the TASE both the cooling capacity and COP decreases. However, for the heat exchangers in both the TASC and TASE it was found that increasing the length initially increases the cooling capacity and COP for all plate spacings but beyond 30 mm they begin to plateau and eventually decrease. Then various combinations were evaluated in DeltaEC to determine the optimal dimensions that would maximise the cooling capacity and COP. Using the optimised dimensions, the cooling capacity improved significantly from 106.38 W to 633.80 W, and the COP from 6.93% to 23.70%.

1. Introduction

A heat-driven thermoacoustic Stirling cryocooler uses a heat input to produce a sound wave which is equivalent to the pressure wave that would be created by a mechanical compressor in a traditional Stirling cryocooler. Hence, these cryocoolers are expected to be simple, lightweight, and highly reliable due to the elimination of all moving mechanical parts which are desirable characteristics for aerospace applications. In this system, a thermoacoustic Stirling engine (TASE) is coupled with a thermoacoustic Stirling cryocooler (TASC). The TASE thermoacoustic core consists of a regenerator placed between an ambient and hot heat exchanger, whilst the TASC core includes an ambient and cold heat exchanger.



The regenerators undergo a thermodynamic cycle synonymous to the Stirling cycle due to the temperature gradient set up across it by heat transfer with the gas, and from the adjacent heat exchangers. In the TASE, the regenerator generates acoustic work from the heat input supplied to the hot heat exchanger which is then passed to the regenerator in the TASC, where the work is converted into thermal energy through the reverse Stirling cycle to produce cooling.

Ueda [1] was the first to create such a machine in 2004, which followed the design of a TASE developed by Backhaus and Swift comprising of a looped tube attached to a straight resonator. However, their design also included a second regenerator sandwiched between two heat exchangers in the looped tube which absorbed the acoustic work from the engine to generate cooling. Although significant improvements have been made over the last two decades, the length of the resonator still poses limitations when it comes to reducing the size of the cryocooler, and therefore small-scale versions of these machines have not seen much success. Thus, this paper aims to develop and optimise a numerical model of a small-scale travelling-wave TASC to assess whether these machines can be made more compact and still achieve high performance.

2. Numerical model & results

2.1 DeltaEC model overview and setup

Thermoacoustic modelling allows for better understanding of the workings of thermoacoustic machines by analysing distributions of acoustic and thermal properties, and how these are influenced by variations in both geometrical and operating parameters. The numerical simulation software 'DeltaEC' was used for this purpose to allow for assessment of various designs and optimisation of parameters for enhanced performance without the issues of time and cost associated with building experimental models. The software uses Rott's linear acoustic theory to numerically integrate the wave equation through user-defined segments (such as ducts, or heat exchangers) in the axial direction, and match the pressures, volume flow rates, and temperatures at the connections between them. Also, for regenerators and stacks the mean temperature profile is determined by the energy equation, using flows at adjacent heat exchangers. DeltaEC incorporates a 'shooting' algorithm whereby the user must enter an equal number of guesses and targets, and it will aim to meet those target values by altering the initial guess values.

The DeltaEC model is comprised of four main sections, two thermoacoustic cores (engine and cooler), the looped tube, and resonator. The segments in the cooler core include an ambient heat exchanger, regenerator, and cold heat exchanger, the same segments are included in the engine core except that it has a hot heat exchanger. All of the heat exchangers are modelled as the parallel-plate type, and the regenerator as a stack of stainless-steel mesh screens. A more detailed explanation of the development of this DeltaEC model and the setup for the numerical simulations can be found elsewhere [2].

2.2 Numerical simulation and results

The aim for this model was to achieve a diameter of 25 mm by reducing the area of the components of a larger TASE DeltaEC model (which had a diameter of 100 mm) that was used as the basis to build this model. Table 1 below provides the key dimensions for all of the components, and it can be seen that the desired area was achieved for all components except the resonator, corresponding to a reduction of 93.8% and 91% for the resonator. Also, the lengths of the resonator and looped tube from the original model were 5.4550 m and 1.3590 m respectively, hence a reduction of 63.3% and 63.2% were achieved for these components. After achieving these reductions, the lengths of the other components (i.e., heat exchangers, regenerators, etc) were then minimised.

However, for the operating conditions of $f = 204.28$ Hz, $p_{\text{mean}} = 3.57$ MPa, and helium as the working gas it was found that the cold heat exchanger of the TASC only achieved a cooling capacity of 106.38 W for a heat input of 1534.5 W at the hot heat exchanger of the TASE, indicating that the dimensions were not optimal and further investigation into the relationship between the geometry and performance was required.

Table 1. List of Dimensions of Components in Small-Scale TASC DeltaEC Model

Segment No.	Component	Dimensions	Size Reduction
2	Looped Tube	$A = 4.9087 \times 10^{-4} \text{ m}^2$ $L_{\text{half}} = 0.5 \text{ m}$	$A = 93.8\%$ $L_{\text{half}} = 63.2\%$
5	TASC Ambient Heat Exchanger	$A = 4.9087 \times 10^{-4} \text{ m}^2$ $L = 2.0 \times 10^{-2} \text{ m}$ $L_{\text{space}} = 1.5 \times 10^{-4} \text{ m}$	$A = 93.8\%$
6	TASC Regenerator	$A = 4.9087 \times 10^{-4} \text{ m}^2$ $L = 2.5 \times 10^{-2} \text{ m}$ Mesh No. = 120 $d_{\text{wire}} = 2.5 \times 10^{-5} \text{ m}$	$A = 93.8\%$
7	TASC Cold Heat Exchanger	$A = 4.9087 \times 10^{-4} \text{ m}^2$ $L = 2.0 \times 10^{-2} \text{ m}$ $L_{\text{space}} = 1.5 \times 10^{-4} \text{ m}$	$A = 93.8\%$
8	TASC Thermal Buffer Tube	$A = 4.9087 \times 10^{-4} \text{ m}^2$ $L = 1.8 \times 10^{-2} \text{ m}$	$A = 93.8\%$
9	TASE Ambient Heat Exchanger	$A = 4.9087 \times 10^{-4} \text{ m}^2$ $L = 2.5 \times 10^{-2} \text{ m}$ $L_{\text{space}} = 1.5 \times 10^{-4} \text{ m}$	$A = 93.8\%$
10	TASE Regenerator	$A = 4.9087 \times 10^{-4} \text{ m}^2$ $L = 2.5 \times 10^{-2} \text{ m}$ Mesh No. = 200 $d_{\text{wire}} = 2.5 \times 10^{-5} \text{ m}$	$A = 93.8\%$
11	TASE Hot Heat Exchanger	$A = 4.9087 \times 10^{-4} \text{ m}^2$ $L = 1.5 \times 10^{-2} \text{ m}$ $L_{\text{space}} = 1.5 \times 10^{-4} \text{ m}$	$A = 93.8\%$
12	TASE Thermal Buffer Tube	$A = 4.9087 \times 10^{-4} \text{ m}^2$ $L = 5.4 \times 10^{-2} \text{ m}$	$A = 93.8\%$
16	Resonator	$A = 7.00 \times 10^{-4} \text{ m}^2$ $L = 2.0 \text{ m}$	$A = 91\%$ $L = 63.3\%$

3. Optimisation

After obtaining the working model of the small-scale TASC in DeltaEC, optimisation of geometric parameters for the regenerators and heat exchangers was conducted to determine the dimensions that would provide maximum cooling capacity and COP for the machine.

3.1 Optimisation of TASC and TASE regenerators

For the regenerators the wire diameter was varied for a range of mesh numbers (60, 100, 120, 150, 200, 300), and then plotted against the cooling capacity and COP, with Figure 1 below showing plots for the cooling capacity of the TASC and TASE, and Figure 2 showing the plots for the COP.

Figure 1 shows that increasing the wire diameter reduces the cooling capacity for both the TASC and TASE regardless of the mesh number. However, the reduction for the TASE is much smaller across the range of wire diameters except for a mesh number of 200 where it drastically decreases for wire diameters greater than 0.04 m. Also, it can be inferred that a mesh number between 100 – 150 provides the maximum cooling capacity for the TASC, and a mesh number between 120 – 150 gives the maximum cooling capacity for the TASE.

Figure 2 illustrates that the COP follows the same downward trend as that of Figure 1 for the TASC, although for a wire diameter greater than 0.04 m a mesh number of 100 gives a better value than a mesh number of 200. For the TASE though, the trends for the COP differ slightly as for the mesh numbers 100 and 120 the COP slightly increases with increasing wire diameter until 0.04 m, where it then starts decreasing, but for mesh numbers higher than 120 the COP only decreases ever so slightly across the range. It is also clear that for a mesh number of 200 the COP significantly decreases for wire diameters greater than 0.04 m. Furthermore, it can be seen that a mesh number between 120 – 150 gives the highest COP for the TASC, and a mesh number between 150 – 200 gives the highest COP for the TASE, although for wire diameters greater than 0.045 m a mesh number of 120 gives a better value than a mesh number of 200.

Using these results, a variety of combinations were evaluated in DeltaEC to determine which would provide the maximum cooling capacity and COP for the entire machine. This resulted in a wire diameter of 0.025 m and mesh number of 120 for the TASC, and 0.025 m and 200 for the TASE. The reason for the higher optimal mesh number for the TASE is because in an engine a finer mesh provides more surface area for gas interaction, thus improving the heat transferred into acoustic power. For a cooler however, the balance of the geometry and porosity to create the desired temperature difference is more crucial, and this is why the optimal value for the TASC does not match that of the TASE. These updated dimensions increased the cooling capacity from 106.38 W to 322.87 W, which just shows the benefit of understanding how parameters influence the performance so that they can be adjusted to improve the operation of the machine.

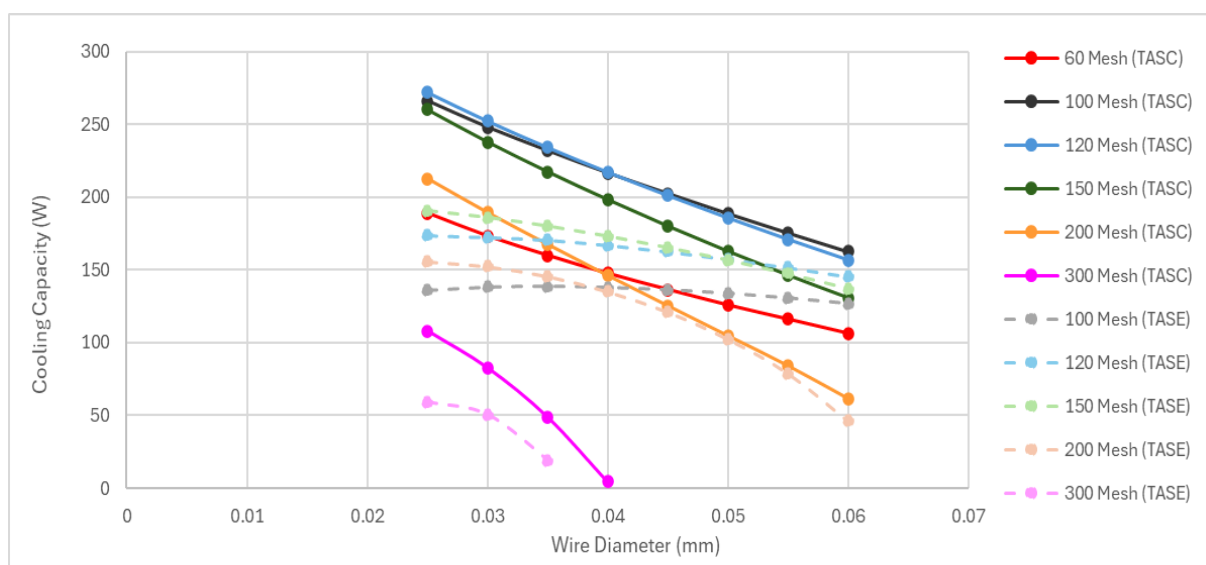


Figure 1. Plots of Cooling Capacity against Wire Diameter for TASC and TASE Regenerator

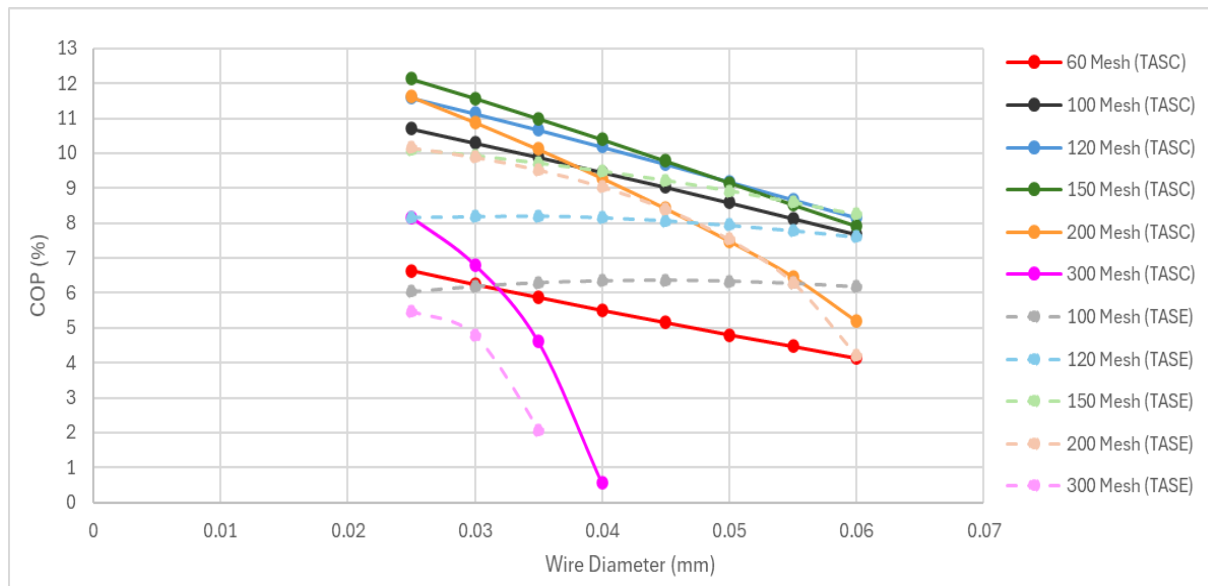


Figure 2. Plots of COP against Wire Diameter for TASC and TASE Regenerator

3.2 Optimisation of heat exchangers

After optimising the regenerators, a similar parameter study was conducted for all of the heat exchangers to explore how the slot width and length affected the cooling capacity and COP of the machine. This involved finding the optimal values for the ambient heat exchanger of the TASC and then using these new dimensions when running the optimisation for the cold heat exchanger. This process continued for the ambient and hot heat exchanger of the TASE, to determine the final dimensions that would provide the maximum cooling capacity and COP. The length was varied for a range of slot widths (2.00×10^{-4} , 4.00×10^{-4} , 6.00×10^{-4} , 8.00×10^{-4}), and then plotted against the cooling capacity and COP, with Figure 3 showing the plots for the ambient and cold heat exchangers of the TASC, and Figure 4 showing the plots for the ambient and hot heat exchangers of the TASE.

3.2.1 TASC heat exchanger results

Figure 3(a) shows that increasing the length initially causes a rapid increase in the cooling capacity for the TASC ambient heat exchanger, but after 30 mm it starts to plateau before starting to decrease. The cold heat exchanger displays similar trends, except that after 30 mm the cooling capacity decreases more rapidly and shifts to a more linear downward trend. Also, it can be seen that for lengths less than 30 mm a slot width of 2.00×10^{-4} m gives the largest cooling capacity for both the heat exchangers, however beyond this point a value of 4.00×10^{-4} m provides slightly more cooling.

Figure 3(b) shows almost identical trends for the ambient heat exchanger as those in Figure 3(a), with the exception that a slot width of 2.00×10^{-4} m provides the greatest cooling capacity until 40 mm. For the cold heat exchanger however, the trends are similar to that of Figure 3(a) until 30 mm, but then the COP decreases much less drastically.

Also, for both the heat exchangers a slot width of 2.00×10^{-4} m gives the highest COP for lengths less than 30 mm, whilst beyond this value a slot width of 4.00×10^{-4} m gives better results.

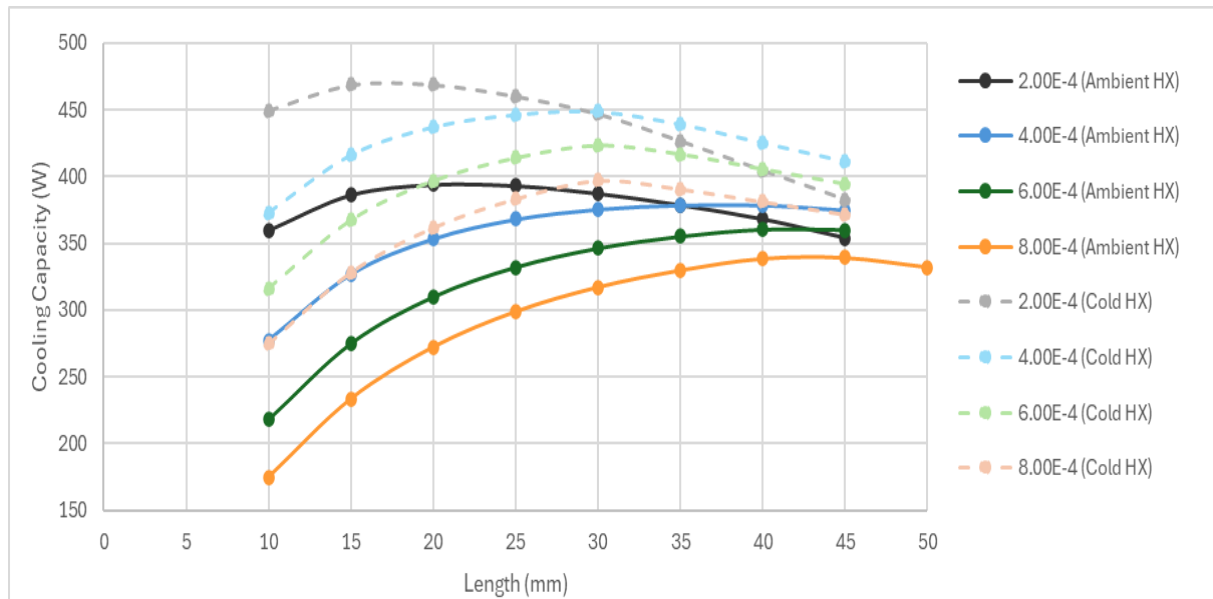


Figure 3(a)

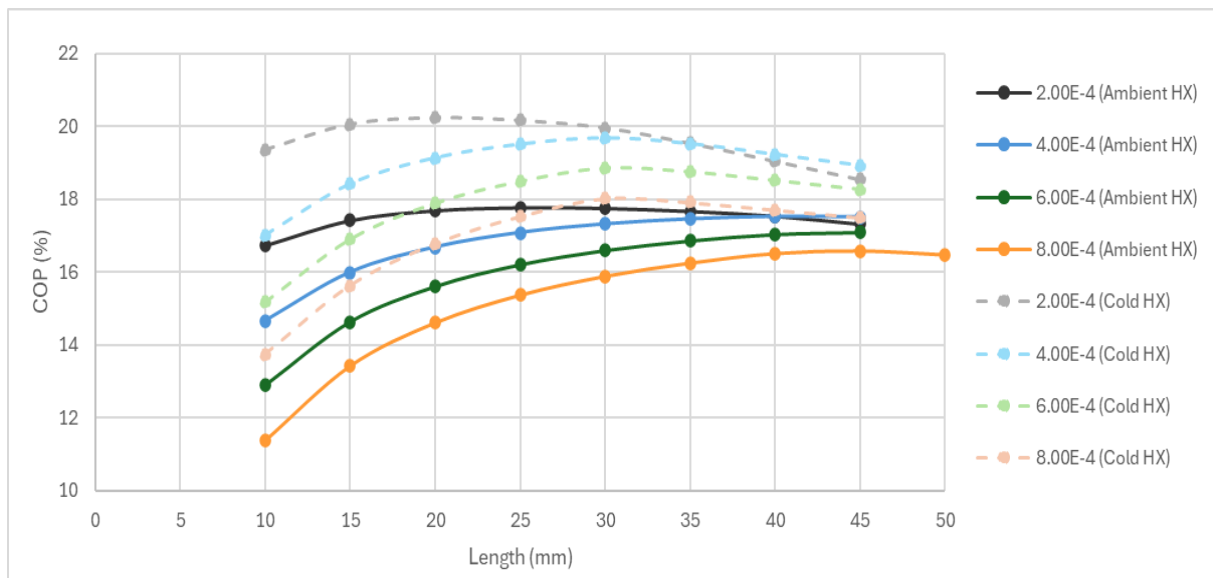


Figure 3(b)

Figure 3. Plots of (a) Cooling Capacity and (b) COP against Length for Ambient and Cold Heat Exchangers of the TASC

3.2.2 TASE heat exchanger results

Figure 4(a) displays similar trends for the TASE ambient heat exchanger as those of the TASC in Figure 3(a), except that a slot width of 2.00×10^{-4} m provides the greatest cooling capacity for the entire range of lengths. Also, similar trends can be seen for the hot heat exchanger for slot widths greater than 2.00×10^{-4} m. However, for a slot width of 2.00×10^{-4} m the cooling capacity is initially unchanged between 10 – 15 mm, and then decreases linearly with further increase in length.

Furthermore, for the hot heat exchanger the values of cooling capacity are significantly higher for a slot width of 2.00×10^{-4} m reaching nearly 850 W, whilst the highest value achieved by the ambient heat exchanger is less than 750 W for a slot width of 4.00×10^{-4} m.

Figure 4(b) presents very similar trends for the COP as those of Figure 4(a) for the ambient heat exchanger, but with a much smaller gradient and after 30 mm the trends also decrease at a significantly slower rate. However, just like in Figure 4(a) a slot width of 2.00×10^{-4} m gives the highest COP for the entire range of lengths. Interestingly, the hot heat exchanger displays trends for the COP that do not match those of the other heat exchangers, whereby increasing the length results in only a slight decrease in the COP for the entire range of lengths. Furthermore, it seems that the trends are reversed such that a slot width of 8.00×10^{-4} m provides the highest COP and a slot width of 2.00×10^{-4} m gives the lowest.

Using these results, the final dimensions for all of the heat exchangers were determined, and the DeltaEC model was run with updated dimensions for both the regenerators and the heat exchangers of the TASC and TASE. After making these adjustments, the cooling capacity nearly doubled, increasing from 322.87 W to 633.80 W.

For the operating conditions of $f = 206.20$ Hz, $Q_{in} = 2674.0$ W, $p_{mean} = 4.95$ MPa, and helium as the working gas, the cold heat exchanger reached a cold temperature of $T_c = 250$ K, with a cooling capacity of $Q_c = 633.80$ W, for a hot temperature of $T_H = 973$ K at the hot heat exchanger. Also, the ambient heat exchangers were maintained at a temperature of $T_a = 300$ K. This shows significant improvement in the cooling capacity from the initial DeltaEC model, increasing by nearly six times from $Q_c = 106.38$ W.

Hence, this just highlights the value of these numerical modelling tools, as by conducting these parameter studies it was possible to get a better understanding of how the geometric parameters of the components influence the performance of these thermoacoustic machines.

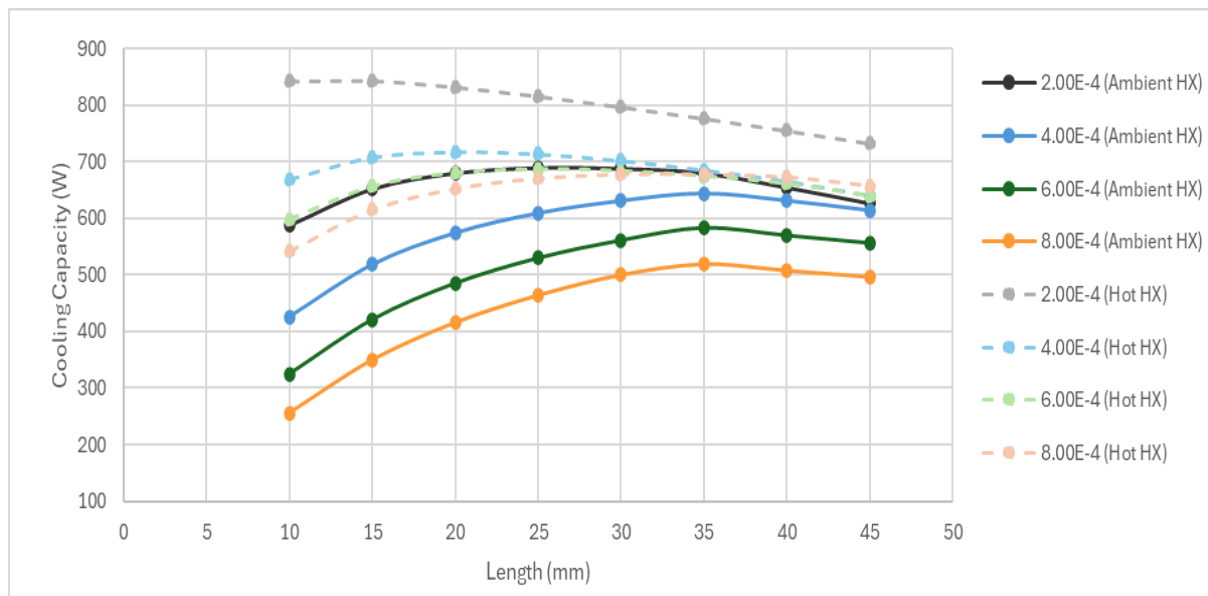


Figure 4(a)

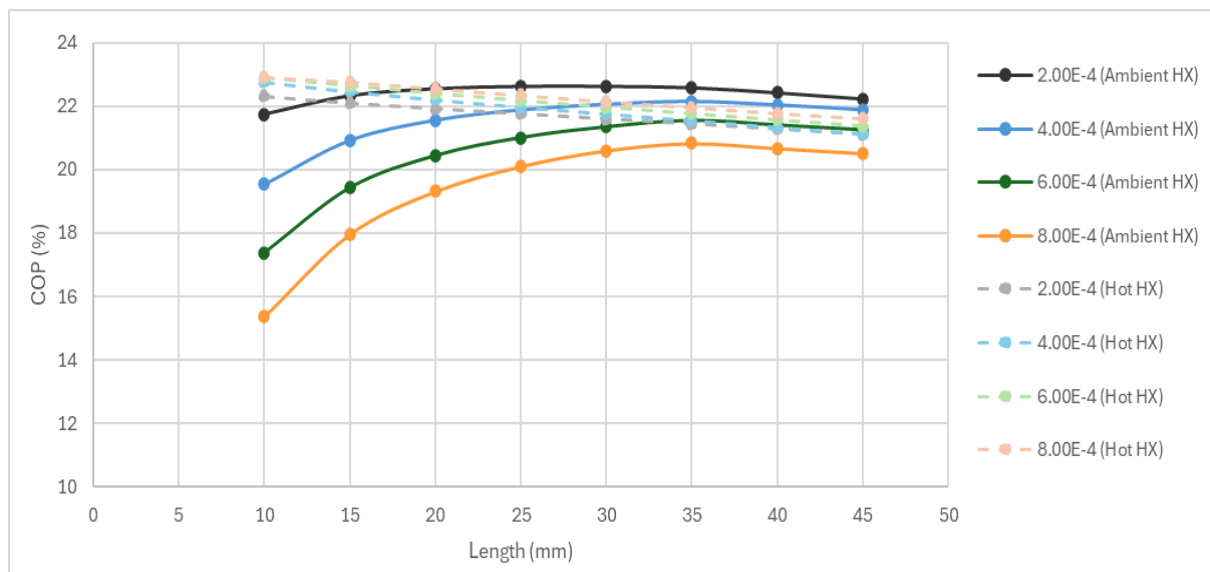


Figure 4(b)

Figure 4. Plots of (a) Cooling Capacity and (b) COP against Length for Ambient and Hot Heat Exchangers of the TASE

4. Conclusions

4.1 Feasibility of small-scale thermoacoustic Stirling cryocoolers

A numerical model of a small-scale travelling wave thermoacoustic Stirling cryocooler (TASC) was developed and optimised in DeltaEC. The optimisation of the regenerators in both the engine and cooler section, provided valuable insight into how the wire diameter and mesh number of the regenerators, showing that increasing the wire diameter decreases both the cooling capacity and COP, regardless of the mesh number.

For the heat exchangers, it was found that increasing the length initially increases the cooling Capacity and COP up to 30 – 40 mm, but beyond this it begins to slowly decrease for all slot widths. Interestingly though, this was not the case for the COP for the hot heat exchanger of the engine as this showed an almost linear downward trend for all slot widths. From these results, the optimal dimensions for the regenerators and heat exchangers were determined which increased the cooling capacity from 106.38 W to 633.80 W.

Overall, the work provided in this paper shows the potential for these thermoacoustic Stirling cryocoolers to be feasible at a small scale which would make them more desirable for applications where both weight and size restrictions are an issue such as aircraft.

Acknowledgments

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References

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